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HEAT TRANSFER IN TURBULENT FLOW OF POLYATOMIC
GASES ALONG A TUBE

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A numerical method is used to calculate the transfer to ammonia for different models of turbulent viscosity. The results obtained are compared with experimental data.

With the increase in heat-flux levels in various powerstations and the growing variety of heat carriers (polyatomic and chemically reacting gases, material in a near-critical and supercritical state, etc.), there has arisen a need for methods of calculation of the turbulent flow along a tube of liquids with strongly variable physical properties.

Both physical and mathematical difficulties beset the solution of this problem. Because there is no consistent theory of turbulence at present, it remains uncertain whether semiempirical models of turbulence developed for flows of incompressible liquid along a tube may be used for liquids with variable properties. From a mathematical viewpoint, the existing temperature and pressure dependences of the physical properties of the liquid lead to "strong" nonlinearity of the initial system of equations, so that it is necessary to use finite-difference methods for its solution.

Such methods have been used to obtain solutions for turbulent flows of gaseous nitrogen and air [1] and hydrogen in a state of equilibrium dissociation [2] along a circular tube.

In [1], a comparative analysis of 11 different models of turbulent viscosity was made on the basis of experimental data. It was shown that in the conditions under consideration the formula of [4] gives the best agreement with experiment [3].

In [2] turbulent viscosity was determined using the Reichardt formula [5] with Goldman's correction [6].

In both cases it was assumed that the turbulent analog of the Prandtl number is unity, that the gas is perfect, and that the pressure dependence of the thermodynamic properties and molecular transfer coefficients is negligible. Note that the last two assumptions considerably restrict the use of these methods, and the conclusions drawn as to their applicability require further verification.

The present paper outlines a finite-difference method that may be used to calculate turbulent flows in circular tubes for arbitrary temperature and pressure dependences of the thermodynamic and transfer properties of the gas.

The results obtained for the heat transfer to ammonia using the formulas of [4, 5] and two variants of the Millionshchikov formula [7, 6] to determine the turbulent viscosity are compared with experimental data [8]. The Millionshchikov formula is of great practical

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TABLE 1. Coefficients in Eq. (8)

φ	a_φ	b_φ	c_φ
u	μ_{ef}	r	0
H	$\frac{\mu}{Pr} + \frac{\mu_\tau}{Pr_\tau}$	0	$4Ek_0 \frac{\partial}{\partial r} \left\{ \left[\mu_{ef} \left(\frac{\mu}{Pr} + \frac{\mu_\tau}{Pr_\tau} \right) \right] r \frac{\partial}{\partial r} \left(\frac{u^2}{2} \right) \right\}$

interest, since it can easily be generalized to the case of flow in channels with roughened walls [7].

The system of equations describing the turbulent flow of liquid in a circular tube is of the form

$$\bar{\rho} \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + 2\bar{\rho} \bar{v} \frac{\partial \bar{u}}{\partial \bar{r}} - \frac{d\bar{p}}{d\bar{x}} + \frac{4}{\bar{r}} \cdot \frac{\partial}{\partial \bar{r}} \left(\bar{\mu}_{ef} \bar{r} \frac{\partial \bar{u}}{\partial \bar{r}} \right), \quad (1)$$

$$\frac{\partial}{\partial \bar{x}} (\bar{\rho} \bar{u} \bar{r}) + 2 \frac{\partial}{\partial \bar{r}} (\bar{\rho} \bar{v} \bar{r}) = 0, \quad (2)$$

$$\bar{\rho} \bar{u} \frac{\partial \bar{H}}{\partial \bar{x}} + 2\bar{\rho} \bar{v} \frac{\partial \bar{H}}{\partial \bar{r}} - \frac{4}{\bar{r}} \cdot \frac{\partial}{\partial \bar{r}} \left\{ \left(\frac{\bar{\mu}}{Pr} + \frac{\bar{\mu}_\tau}{Pr_\tau} \right) \bar{r} \frac{\partial \bar{H}}{\partial \bar{r}} \right\} + Ek_0 \left[\bar{\mu} \left(1 - \frac{1}{Pr} \right) + \bar{\mu}_\tau \left(1 - \frac{1}{Pr_\tau} \right) \right] \bar{r} \frac{\partial}{\partial \bar{r}} \left(\frac{\bar{u}^2}{2} \right), \quad (3)$$

$$\int_0^1 \bar{\rho} \bar{u} \bar{r} d\bar{r} = 1/2, \quad (4)$$

where

$$\bar{u} = \frac{u}{u_m}, \quad \bar{v} = \frac{v Re_0}{u_m}, \quad \bar{p} = \frac{p - p_0}{\rho_0 u_m^2}, \quad \bar{H} = \frac{H}{H_m}, \quad Pr = \frac{\mu c_p}{\lambda},$$

$$\bar{x} = \frac{x}{d Re_0}, \quad \bar{r} = \frac{r}{r_m}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{\mu} = \frac{\mu}{\mu_0}, \quad Re_0 = \frac{\rho_0 u_m d}{\mu_0}, \quad Ek_0 = \frac{u_m^2}{H_m}.$$

The longitudinal velocity component and the enthalpy are referred to their mean-mass values in the initial cross section of the tube:

$$u_m = \frac{2\pi \int_0^{r_w} \rho u_0^2 r dr}{m}, \quad H_m = \frac{2\pi \int_0^{r_w} H_0 \rho u_0 r dr}{m}.$$

The physical properties of the liquid are referred to their value at the pressure p_0 at the tube inlet and the temperature T_0 corresponding to p_0 and H_m .

The boundary conditions for Eqs. (1)-(4) are

$$\bar{u} = \bar{u}_0(\bar{r}), \quad \bar{H} = \bar{H}_0(\bar{r}), \quad \bar{p} = 0 \quad \text{for} \quad \bar{x} = 0, \quad 0 \leq \bar{r} \leq 1; \quad (5)$$

$$\frac{\partial \bar{u}}{\partial \bar{r}} = 0, \quad \frac{\partial \bar{H}}{\partial \bar{r}} = 0 \quad \text{for} \quad \bar{x} > 0, \quad \bar{r} = 0; \quad (6)$$

$$\bar{u} = 0, \quad \bar{v} = 0, \quad \bar{H} = \bar{H}_w(\bar{x}) \quad \text{or} \quad \bar{q} = \frac{\bar{\lambda}}{c_p} \frac{\partial \bar{H}}{\partial \bar{r}} = \bar{q}_w(\bar{x}) \quad \text{for} \quad \bar{x} > 0, \quad \bar{r} = 1, \quad (7)$$

where

$$\bar{\lambda} = \frac{\lambda}{\lambda_0}, \quad \bar{c}_p = \frac{c_p}{c_{p_0}}, \quad \bar{q} = \frac{q c_{p_0} r_w}{\lambda_0 H_m}.$$

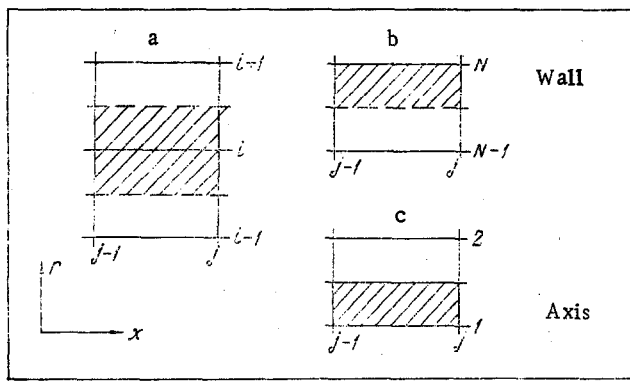


Fig. 1. Position of grid points for internal (a) and boundary (b, c) points of the region.

In what follows the bar above the dimensionless quantities is omitted.

The approach proposed in [9, 10] is adopted for the development of a method of solution of Eqs. (1)-(4) with the boundary conditions in Eqs. (5)-(7). After introducing the current function $\Psi(r) = \int_0^r \rho u dr$, Eqs. (1) and (3) may be written in the single form

$$\frac{\partial}{\partial x} \left(\varphi \frac{\partial \Psi}{\partial r} \right) - \frac{\partial}{\partial r} \left(\varphi \frac{\partial \Psi}{\partial x} \right) - 4 \frac{\partial}{\partial r} \left(a_{\varphi} r \frac{\partial \varphi}{\partial r} \right) + b_{\varphi} \frac{d\varphi}{dx} + c_{\varphi} = 0. \quad (8)$$

Table 1 gives the form of the coefficients a_{φ} , b_{φ} , and c_{φ} for the equations of motion and energy.

A finite-difference approximation of Eq. (8) for points within the region may be obtained using the six-point scheme shown in Fig. 1a.

First, Eq. (8) is integrated over the area shown shaded in Fig. 1a. After taking the integral with first-order accuracy in x and second-order accuracy in r , simple transformations give the result

$$\varphi_i^j = (A_{\varphi})_i \varphi_{i+1}^j + (B_{\varphi})_i \varphi_{i-1}^j + (C_{\varphi})_i + (D_{\varphi})_i + (E_{\varphi})_i p_j. \quad (9)$$

The coefficients A_{φ} , B_{φ} , and C_{φ} for the equations of motion and energy may be represented by the single equation

$$\begin{aligned} (A_{\varphi})_i &= [(k_{i+1}^j + \sigma_1 (m_{\varphi})_{i+\frac{1}{2}}^j) / (\Gamma_{\varphi})_i], \quad (B_{\varphi})_i = [k_{i-1}^j + \sigma_1 (m_{\varphi})_{i-\frac{1}{2}}^j] / (\Gamma_{\varphi})_i; \\ (C_{\varphi})_i &= \{k_i^{j-1} \varphi_i^{j-1} + (1 - \sigma_1) [(n_{\varphi})_{i+\frac{1}{2}}^j + (n_{\varphi})_{i-\frac{1}{2}}^j] / (\Gamma_{\varphi})_i \}; \end{aligned} \quad (10)$$

where

$$\begin{aligned} k_i^{j-1} &= (\Psi_{i+1}^{j-1} - \Psi_{i-1}^{j-1}) / 2; \quad k_{i+\frac{1}{2}}^j = (\Psi_{i+1}^j + \Psi_i^j - \Psi_{i-1}^j - \Psi_i^{j-1}) / 4; \\ k_{i\pm 1}^j &= (|k_{i\pm \frac{1}{2}}^j| \pm k_{i\pm \frac{1}{2}}^j); \quad (\Gamma_{\varphi})_i = k_i^{j-1} + k_{i+1}^j + k_{i-1}^j + \sigma_1 [(m_{\varphi})_{i+\frac{1}{2}}^j + (m_{\varphi})_{i-\frac{1}{2}}^j]; \\ (m_{\varphi})_{i+\frac{1}{2}}^j &= \frac{r_{i+1} + r_i}{r_{i+1} - r_i} \Delta x [(a_{\varphi})_{i+1}^j + (a_{\varphi})_i^j]; \quad (n_{\varphi})_{i+\frac{1}{2}}^j = (m_{\varphi})_{i+\frac{1}{2}}^j (\Psi_{i+1}^{j-1} - \Psi_i^{j-1}), \\ \Delta x &= x_j - x_{j-1}, \quad 0 \leq \sigma_1 \leq 1. \end{aligned}$$

The coefficients D_{φ} and E_{φ} depend on the particular equation: For the equation of motion

$$(D_u)_i = S_i p_{j-1} / 2 / (\Gamma_u)_i, \quad (E_u)_i = -S_i / 2 / (\Gamma_u)_i, \quad (11)$$

where

$$S_i = [(r_i + r_{i+1})^2 - (r_i + r_{i-1})^2] / 4;$$

and for the energy equation

$$(D_H)_i = [\sigma_2 (t_{i+\frac{1}{2}}^j - t_{i-\frac{1}{2}}^j) + (1 - \sigma_2) (t_{i+\frac{1}{2}}^{j-1} - t_{i-\frac{1}{2}}^{j-1})] / (\Gamma_H)_i, \quad (12)$$

TABLE 2. Models of Turbulent Viscosity

Model No	Literature source	$\mu_T \cdot \lambda_T$	η
1	[4]	$\mu_T = \rho \{0.4 (r_w - r) [1 - \exp(-\eta/26)]\}^2 \left \frac{\partial u}{\partial r} \right $ $\lambda_T = c_p \mu_T$	$\eta = \frac{r_w - r}{v_w} \sqrt{\frac{\tau_w}{\rho_w}}$
2	[5] [6]	$\mu_T = \mu \begin{cases} 0.4 \left(\eta - 11 \ln \frac{\eta}{11} \right) & 0 \leq \eta \leq 50, \\ \frac{0.4}{3} \cdot \frac{\eta}{r_w^3} (0.5 r_w^2 + r^2) (r_w - r), & \eta > 50 \end{cases}$ $\lambda_T = c_p \mu_T$	$\mu = \int_r^{r_w} \frac{1}{v} \sqrt{\frac{\tau_w}{\rho}} dr$
3	[7]	$\mu_T = \begin{cases} 0 & \eta \leq \delta^* = 7.8, \\ 0.4 r_w v_w \rho r (\eta - 7.8), & \eta > \delta^*. \end{cases}$ $\lambda_T = \begin{cases} 0 & \eta \leq \delta^* Pr^{-1/4} \\ 0.4 r_w v_w \rho c_p r (\eta - 7.8 Pr^{-1/4}), & \eta > \delta^* Pr^{-1/4} \end{cases}$	$\eta = \frac{r_w - r}{v_w} \sqrt{\frac{\tau_w}{\rho_w}}$
4	[7] [6]	$\mu_T = \begin{cases} 0 & \eta \leq \delta^*, \\ 0.4 r_w v_w \rho r (\eta - 7.8), & \eta > \delta^* \end{cases}$ $\lambda_T = \begin{cases} 0 & \eta \leq \delta^* Pr^{-1/4} \\ 0.4 r_w v_w \rho c_p r (\eta - 7.8 Pr^{-1/4}), & \eta > \delta^* Pr^{-1/4} \end{cases}$	$\eta = \int_r^{r_w} \frac{1}{v} \sqrt{\frac{\tau_w}{\rho}} dr$

$$(E_H)_i = 0, \quad 0 \leq \sigma_2 \leq 1, \tag{12}$$

where

$$t_{i+\frac{1}{2}}^j = \frac{Ek_0 \Delta x}{2} \cdot \frac{r_{i+1} + r_i}{r_{i+1} - r_i} [(a_u)_{i+1}^j + (a_u)_i^j - (a_H)_{i+1}^j - (a_H)_i^j] [(u_{i+\frac{1}{2}}^j)^2 - (u_i^j)^2].$$

The expression for $k_{i\pm 1}^j$ is equivalent to the orientation of the radial derivative in the direction opposite to the flow [9]. The finite-difference representation of the flow-conservation condition in Eq. (4) is

$$\sum_{i=1}^N \rho_i u_i S_i = 1. \tag{13}$$

To close the system of difference equations, the boundary conditions in Eqs. (6) and (7) must be imposed on Eqs. (9) and (13).

If Eq. (8) is integrated over the areas shown shaded in Fig. 1b, c, using the boundary condition in Eq. (6) and the last condition in Eq. (7), respectively, with the same accuracy as for the internal points of the region, the result obtained is

$$\varphi_1^j = (A_\varphi)_1 \varphi_2^j + (C_\varphi)_1 + (D_\varphi)_1 + (E_\varphi)_1 p_j, \tag{14}$$

$$H_{N+1}^j = (B_H)_{N+1} H_N^j + (C_H)_{N+1} + (D_H)_{N+1}. \tag{15}$$

The coefficients in Eqs. (14) and (15) may be calculated from Eqs. (10)-(12) after discarding terms containing the subscripts $i - 1$ and $i - 1/2$ or $i + 1$ and $i + 1/2$ and replacing the subscript i by 1 or $N + 1$, respectively. The coefficients k_1^{j-1} , k_{N+1}^{j-1} , and $(D_H)_{N+1}$ form an exception; in this case, the appropriate expressions are

$$k_1^{j-1} = (\Psi_2^{j-1} - \Psi_1^{j-1})/2, \quad k_{N+1}^{j-1} = (\Psi_{N+1}^{j-1} - \Psi_N^{j-1})/2,$$

$$(D_H)_{N+1} = [\sigma_2 (f_j - t_{N+\frac{1}{2}}^j) + (1 - \sigma_2) (f_{j-1} - t_{N+\frac{1}{2}}^{j-1})] / (\Gamma_H)_{N+1},$$

where

$$f_j = 4 (q_w)_j / Pr_0; \quad Pr_0 = \frac{\mu_0 c_{p0}}{\lambda_0}.$$

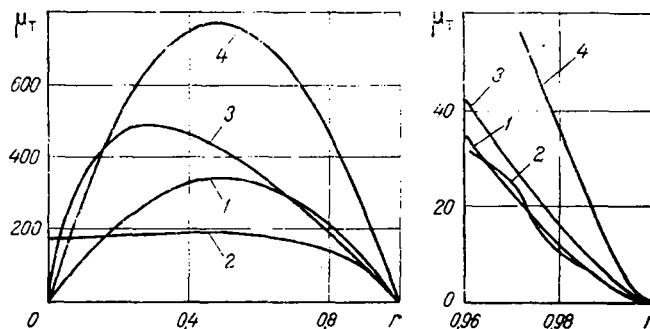


Fig. 2. Dimensionless profile of turbulent viscosity for conditions No. 6 with $x/d = 80$ (curves 1-4 correspond to the models in Table 2).

Specifying the boundary conditions of the first kind in Eq. (7) at the wall, they may be replaced directly by their difference analogs

$$u_{N+1} = 0, H_{N+1} = H_w. \quad (16)$$

In fact, Eqs. (9) and (13) together with Eqs. (14), (16) or (15) form a nonlinear algebraic system in terms of the enthalpy, velocity, and pressure at the j -th layer, which is solved by successive approximation.

The calculation begins with the solution of the energy equation by the trial-and-error method. The coefficients of Eqs. (9), (14) (15), or (16) are calculated from the values of the unknown functions obtained in the preceding iteration. The parameters from the preceding step in x are used as the initial approximation. The velocity profile, the current function, and the pressure are then calculated, using Eqs. (9), (13), (14), and (16), preliminarily converted to the form

$$u_i = G_i + F_i p, \quad p = \frac{1 - \sum_{i=1}^N \rho_i S_i G_i}{\sum_{i=1}^N \rho_i S_i F_i} \quad (17)$$

The coefficients G_i and F_i in Eq. (17) are determined using the following recurrence relations:

$$G_i = K_i G_{i+1} + L_i, \quad F_i = K_i F_{i+1} + M_i, \quad i = 1, 2, \dots, N-1, \\ G_N = L_N, \quad G_{N+1} = 0, \quad F_N = M_N, \quad F_{N+1} = 0,$$

where

$$K_i = (A_u)_i / [1 - (B_u)_i K_{i-1}], \quad L_i = [(B_u)_i L_{i-1} + (C_u)_i + (D_u)_i] / [1 - (B_u)_i K_{i-1}], \\ M_i = [(B_u)_i M_{i-1} + (E_u)_i] / [1 - (B_u)_i K_{i-1}], \quad i = 2, 3, \dots, N, \\ K_1 = (A_u)_1, \quad L_1 = (C_u)_1 + (D_u)_1, \quad M_1 = (E_u)_1.$$

As in the case of the energy equation, the coefficients A_u , B_u , C_u , D_u , and E_u are determined from the values of the unknown functions in the preceding iteration.

From the known values of the total enthalpy and the velocity, the enthalpy and temperature profiles are calculated. Further, the results obtained for the pressure and temperature are used to determine the thermodynamic properties and molecular transfer coefficients. The effective viscosity and heat conduction are calculated in accordance with the model of turbulent viscosity adopted. This procedure is repeated until convergence is obtained.

This method is used to calculate the turbulent flow of ammonia for four models of turbulent viscosity (Table 2). Tabulated values are taken for the properties of ammonia [11].

The grid employed has a constant increment in x ($\Delta x = 0.5$) and nonuniform divisions along the r axis (the total number of points in the transverse direction is 50). The values of σ_1 and σ_2 chosen were: $\sigma_1 = 1$; $\sigma_2 = 0.5$.

In the initial tube cross section (at $x = 0$) a homogeneous enthalpy profile and a completely developed velocity profile are specified for the isothermal turbulent flow in the

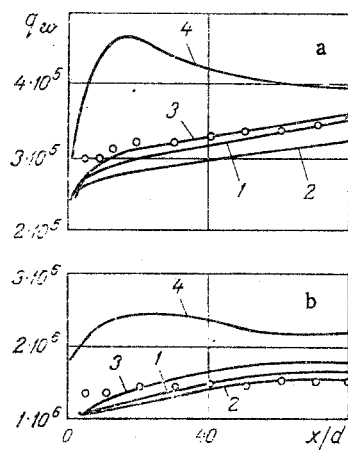


Fig. 3

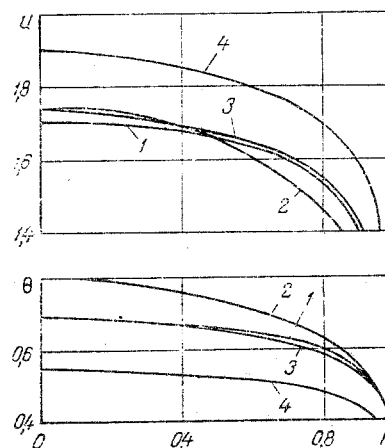


Fig. 4

Fig. 3. Distribution of heat flux q_w , W/m^2 , over the length of the tube: The points give experimental results [8]; curves 1-4 are as in Fig. 2; a) conditions No. 6; b) No. 26.

Fig. 4. Dimensionless velocity and temperature profile for conditions No. 6 with $x/d = 80$; notation as in Fig. 2.

tube; this corresponds to the experimental conditions. The experimental distribution $T = T_w(x)$ is taken as the boundary condition at the wall for the energy equation. Because there are no experimental data for $x/d < 3$, an arbitrary boundary condition for this region is taken in the calculations. Thus, the comparison of the calculated results with the experimental data should begin a certain distance from the inlet. Preliminary calculations [with different expressions for $T_w(x)$ on the section $x/d < 3$] show that this distance does not exceed six diameters. No more than four iterations are required to obtain agreement with accuracy up to 0.2% between successive approximations. The integral thermal-balance condition over the increment Δx is then satisfied with accuracy $\pm 0.15\%$.

Results obtained for two characteristic flow conditions for ammonia — No. 6 ($Re_0 = 1.2 \cdot 10^5$, $T_w/T_l = 1.4$) and No. 26 ($Re_0 = 1.3 \cdot 10^6$, $T_w/T_l = 1.9$) [8] — are shown in Figs. 2-4.

As follows from Fig. 2, in the region at the wall all the values of the viscosity, except those corresponding to model No. 4, are close. The turbulent viscosity profiles calculated by the Van Driest and Millionshchikov formulas reach a maximum in the central flow region and vanish at the tube axis. The viscosity profile calculated by the Reichardt formula is qualitatively different and retains a practically constant value in the flow core right up to the tube axis.

The velocity and temperature distributions over the tube cross section (Fig. 4) depend significantly on the model of the turbulent viscosity adopted. However, for the first three models, the difference in the values of the heat flux at the wall $q_w(x)$ shown in Fig. 3 is small and there is good agreement with the experimental data (shown by points in Fig. 3).

The results obtained indicate that the local characteristics of the flow are very sensitive to the distribution of turbulent viscosity over the whole flow region. The effective viscosity in the region at the wall has the main effect on the integral flow characteristics.

The Van Driest model, the Reichardt model with Goldman's correction, and the Millionshchikov model give satisfactory results for the heat transfer to polyatomic gases with significant temperature and pressure dependences of the thermophysical and transfer properties.

NOTATION

x , r , coordinates; r_w and d , radius and diameter of tube; v , transverse velocity component; ρ , density; μ , dynamic viscosity; λ , heat conduction; c_p , specific heat at constant pressure; T_l , mean-calorimetric temperature of liquid; T_w , temperature of tube wall; $\theta = (T - T_w)/(T_l - T_w)$; τ_w , friction at wall; q_w , heat flux at wall; i , point number along the r axis; j , layer number over x ; N , number of intervals along the r axis; ν , kinematic viscosity. Indices: ef, effective value; T, turbulent component; 0, value at inlet cross section of channel; w, value at channel wall.

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STEADY THREE-DIMENSIONAL TEMPERATURE FIELD IN COOLED
TURBINE BLADES

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A method based on the use of Green functions is proposed for calculating the temperature field of cooled turbine blades. The method presumes the use of high-speed computers with large memories.

The creation of stoichiometric gas turbine engines and installations requires the solution of complex Scientific-technical problems. One of them is the reliable detailed calculation of the three-dimensional fields of temperatures and stresses in cooled turbine blades.

With an increase in the gas temperatures and intensification of the cooling the temperature gradients increase both over the height of the blades (especially in the basal zone) and through the cross section (especially in the zone of the edges and perforations). Under these conditions solutions based on the separation of the three-dimensional problem into one-dimensional and two-dimensional problems [1, 2] can lead to considerable errors.

An approximate solution of the three-dimensional problem of steady heat conduction in application to turbine blades with open cooling, reduced to the stage of practical use in contrast to [3], is presented in the present report. Such a problem comes down to integration in a simply connected region (the body of the blade) surrounded by a continuous medium (the gas and the coolant) with locally varying parameters: the temperature $T_{\text{sur.med}}^*$ (from T_g^* to T_{cool}^*) and the heat-transfer coefficients α (from α_g on the gas side to α_{cool} on the coolant side).

The solution described below is also valid for blades with a closed cooling system (a multiply connected region). The method presumes the use of a computer.

In the first approximation we solve the three-dimensional problem of heat conduction with $\lambda(T) = \text{const}$, i.e., the system

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0, \quad (1)$$

$$\alpha (T_{\text{sur.med}}^* - T_s) dF = \lambda \left. \frac{\partial T_s}{\partial n_s} \right|_{n_s=0} dF. \quad (2)$$

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